SIMPLE CONNECTIVITY OF RANDOM SIMPLICIAL COMPLEXES VIA TRIADIC PROCESS PROPAGATION

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When does a random 2-dimensional simplicial complex become simply connected?

Several yeras ago, Linial and Meshulam introduced $Y_d(n, p)$, a random model of simplicial complexes that generalizes the classical random graph model of Erdös and Rényi. It has a full $(d-1)$-dimensional skeleton on n vertices, and every d-face is added independently with probability p. The fundamental group of $Y_2(n, p)$ was first studied by Babson, Hoffman and Kahle, who showed that if $p < n^\alpha$ for $\alpha > 1/2$ then the fundamental group is nontrivial with high probability. On the other hand, they proved that Y is simply connected $p > \sqrt{4 \log n/n}$.

In this talk, we show how to improve their upper bound and prove that if $p = c/\sqrt{n}$ for $c > 1/2$, then $Y_2(n, p)$ is simply-connected with high probability. Moreover, we show that in this regime the random complex contains a collapsible subcomplex with a full 1-skeleton.

For this purpose, we introduce and analyse the following random graph process. We start with the star graph on the same vertex set, containing all the edges incident to some vertex v, and repeatedly add an edge xy if there is a vertex z such that xz and zy are already in the graph and the face xzy belongs to Y. Our analysis relies on the differential equations method for random graph processes, and especially on the tools which were developed in the analysis of the triangle-free process by Bohman.

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