

In the talk, I will present the basic properties and characterizations of so-called “sweakly modular graphs”, of their associated cell complexes, and of groups acting on them. According to the urbain dictionary <http://fr.urbandictionary.com/>, “sweak” is a hybrid of “super” and “weak”. And indeed, sweakly modular graphs represent a subclass of weakly modular graphs and are a common generalization of strongly modular and dual polar graphs. This research is a part of our general program of investigation of “nonpositive curvature” and “local-to-global” properties and characterizations of weakly modular graphs and their subclasses, and the talk is based on the paper J. Chalopin, V. Chepoi, H. Hirai, D. Osajda, Weakly modular graphs and nonpositive curvature, arXiv:1409.3892.

Projective and polar spaces are the most fundamental types of incidence geometries. *Polar spaces* (introduced and classified by J. Tits, 1974) are essentially point-line geometries Π in which for any line ℓ and any point $p \notin \ell$, p is either collinear with a single point of ℓ or p is collinear with all points of ℓ . A *dual polar space* (or a *dual polar graph*) Π^* is a point-line geometry whose points are the maximal subspaces of Π (they all have the same dimension n) and lines are the pairs of such subspaces which intersect in a subspace of dimension $n - 1$. P. Cameron (1982) characterized all dual polar graphs. Revisiting this characterization, we prove that the dual polar graphs are exactly the thick (each pair of vertices at distance 2 is contained in a square) weakly modular graphs not containing two forbidden isometric subgraphs: K_4^- and $K_{3,3}^-$. Furthermore, we present a local-to-global characterization of dual polar graphs G by replacing weak modularity (a global metric condition) by simple connectivity of the triangle-square complex of G and local conditions. This implies and provides an alternative proof of a (difficult) result by A. E. Brouwer and A. M. Cohen (1986).

Now, a *sweakly modular graph* (swm-graph for short) is a weakly modular graph not containing isometric K_4^- and $K_{3,3}^-$ (i.e., the difference with dual polar graphs is that their are not thick). Bipartite swm-graphs are exactly the orientable modular graphs, which have been investigated previously by H. Hirai (2014) and generalize median graphs (1-skeleta of CAT(0) cube complexes). We show that any swm-graph G is obtained by amalgamating dual polar subgraphs of G ; each such dual polar subgraph is the gated hull $\langle\langle x, y \rangle\rangle$ of a Boolean pair x, y of G , i.e., of a pair of vertices x, y lying on an isometric cycle of G . We show that the *orthoscheme complex* $K(G)$ of an swm-graph G – obtained as the union of all orthoschemes of the gated dual polar subgraphs of G – has many nice properties: (a) $K(G)$ is contractible, (b) $K(G)$ is injective with respect to the ℓ_∞ -metric, (c) $K(G)$ is strongly modular with respect to the ℓ_1 -metric, and (d) $K(G)$ is CAT(0) with respect to the ℓ_2 -metric in several important cases and we conjecture that $K(G)$ is always CAT(0). Finally, using the fact that the thickening G^Δ of any swm-graph G is a Helly graph and the fact that in G one can define locally-recognizable normal Boolean-gated paths, we can prove that sweakly modular groups (i.e., groups acting geometrically on swm-graphs) are biautomatic. This generalizes a result by Niblo and Reeves (1998) for CAT(0) cube complexes.